PHYSICS NYB-10/11 Winter 2007

Lecture 3: The Electric Field

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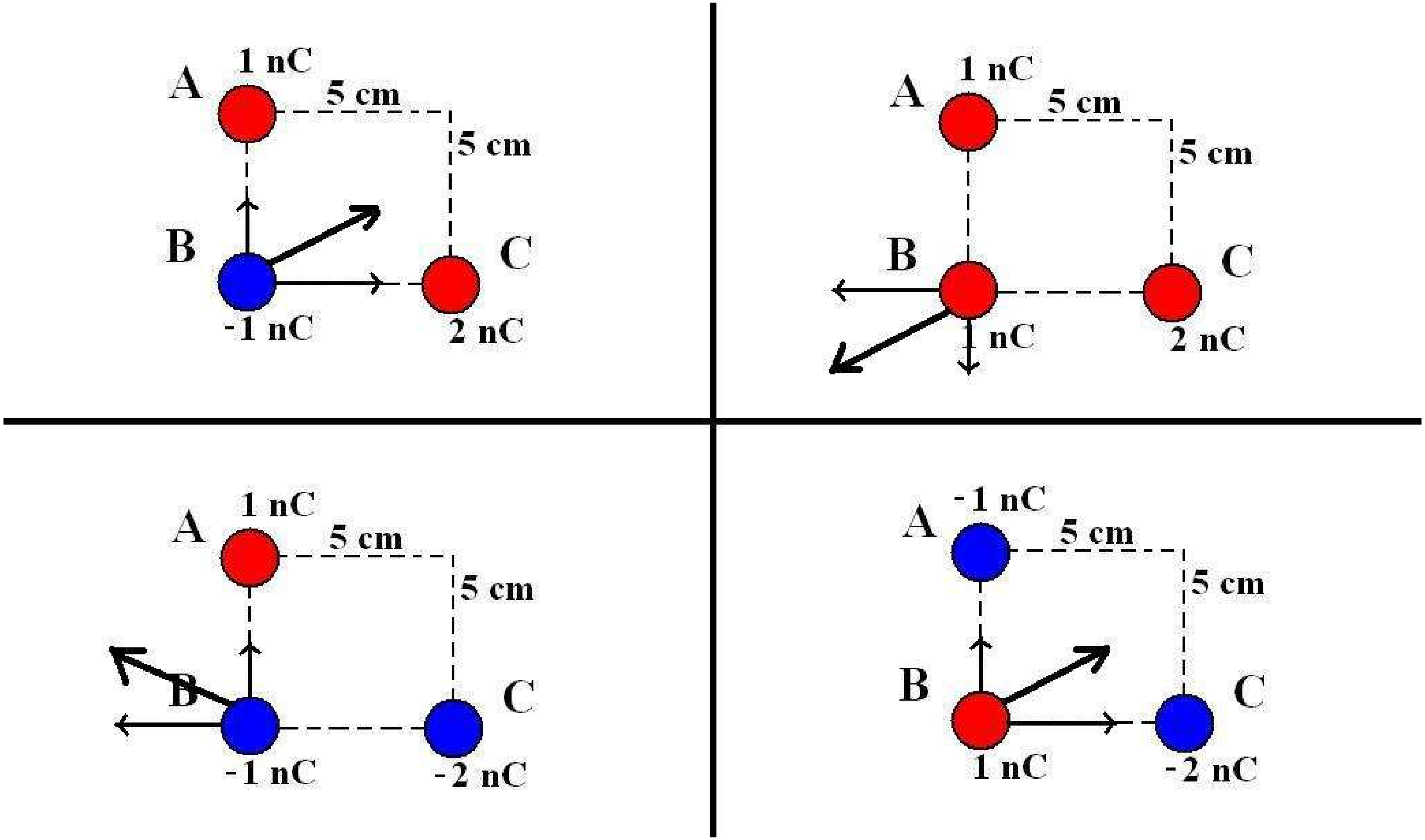
Review

- The magnitude of the electric force between two point charges q_1 and q_2 is given by Coulomb's law $|\vec{F}_e| = k_e \frac{|q_1||q_2|}{r^2}$.
- The force is repulsive for like charges and attractive for opposite charges.
- The force is directed along the line joining the two charges.
- The force acts *on both charges*, and is equal and opposite.
- The net force from many charges is the sum of the forces from each of them. This is called the *superposition principle*.

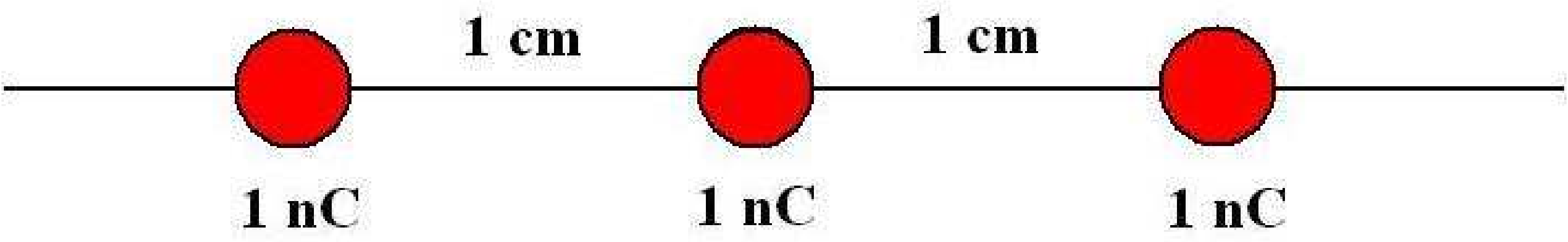
Review

In each of the cases below, draw to scale the forces on charge B from charges A and C, as well as the resultant force.



Review

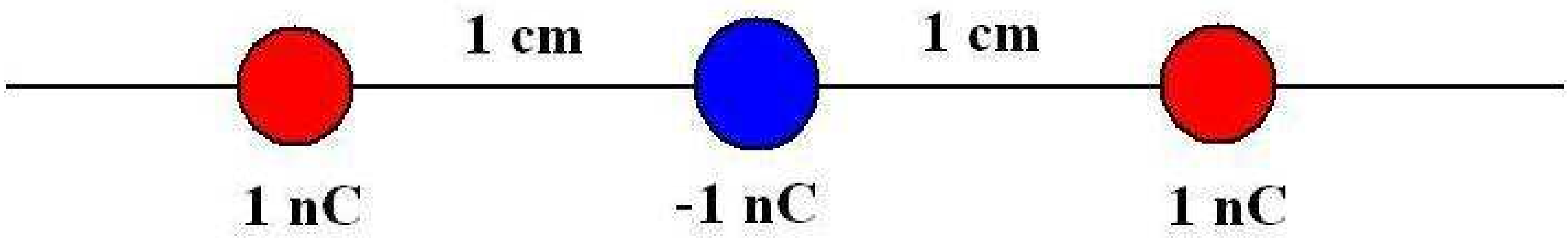
In the situation pictured here, is the middle charge in equilibrium? If yes, is it a stable or unstable equilibrium, and if not, where should it be placed to be at equilibrium?



The forces from the two other charges are equal, since they are the same distance away and have the same charge. They are in opposite directions, so they cancel. This means the middle charge *is* in equilibrium. To figure out if this is stable, think of moving the middle charge slightly to one side. Then the charge it is now closer to will repel it more strongly, and the one it is farther from will repel it less strongly. This will push it back towards its equilibrium point, so that the equilibrium is a stable one.

Review

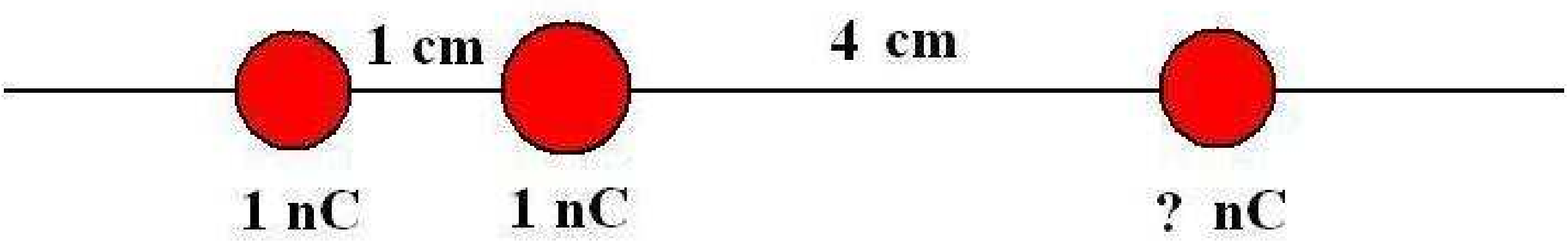
In the situation pictured here, is the middle charge in equilibrium? If yes, is it a stable or unstable equilibrium, and if not, where should it be placed to be at equilibrium?



The forces from the two other charges are equal, since they are the same distance away and have the same charge. They are in opposite directions, so they cancel. This means the middle charge *is* in equilibrium. To figure out if this is stable, think of moving the middle charge slightly to one side. Then the charge it is now closer to will attract it more strongly, and the one it is farther from will attract it less strongly. This will pull it away from its equilibrium point, so that the equilibrium is an *unstable* one.

Review

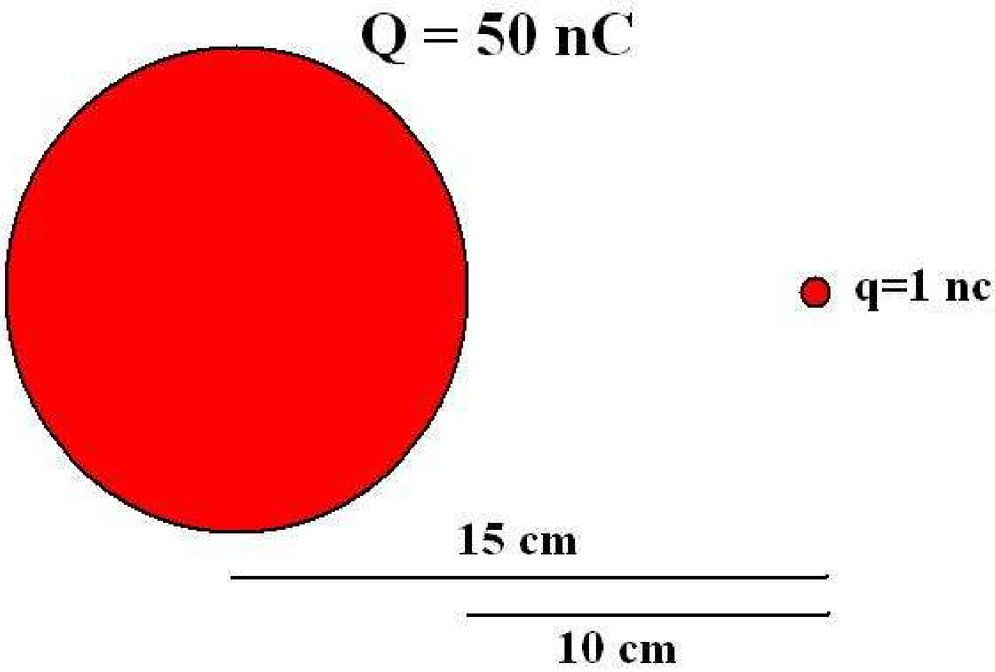
In the situation pictured here, what should value of the right-most charge be for the middle charge to be at equilibrium? Will this be a stable equilibrium?



The left charge is four times closer than the right charge, which leads to a force sixteen times larger, if the left and right charges were equal. For the forces to be equal, the charge on the right one must therefore be sixteen times larger than on the left one, so $q = 16$ nC. It must be a positive charge, so its force points in the opposite direction than the force from the left charge. For the same reasons as in the previous questions, this will be a stable equilibrium.

Review

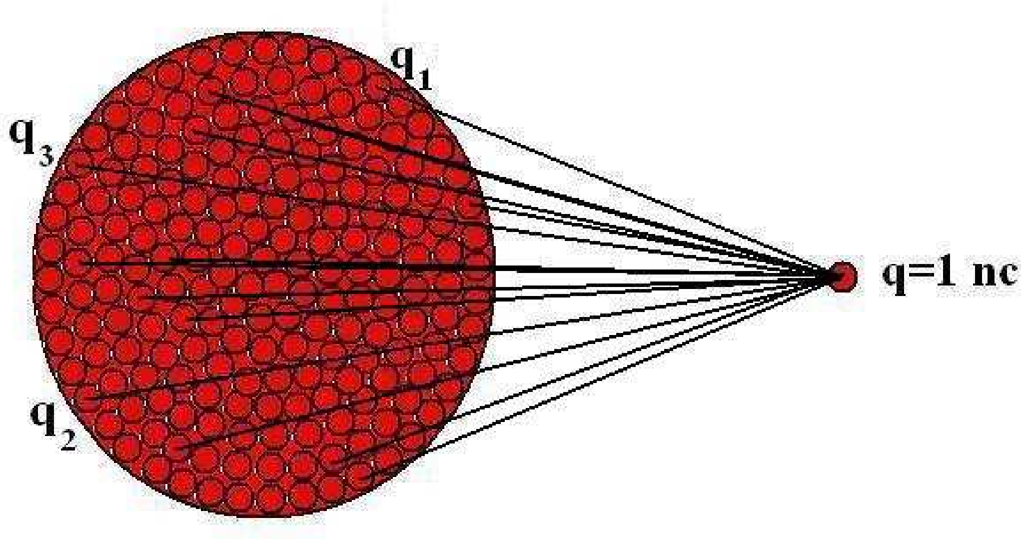
What is the magnitude and direction of the force between the charges pictured here?



Actually, this is a trick question... You can't answer this, since Coulomb's law applies between *point charges*, which is not the case here!

Review

However, a large charged object is simply a collection of point charges.



$$F_1 + F_2 + F_3 + ... = F_{tot}$$

We know what to do when multiple charges are present: we add the forces of each individual charge to get the net force. We will in a few weeks learn a systematic way of doing this.

Coulomb's law and gravity

You might or might not have seen that the law of universal gravitation has the same form as Coulomb's law for the electric force

$$|\vec{F}_g| = G\frac{m_1 m_2}{r^2}$$

where $G = 6.673 \times 10^{-11}\text{Nm}^2/\text{kg}^2$ is Newton's constant.

Question: can you spot one *fundamental* difference between the law of gravitation and Coulomb's law?

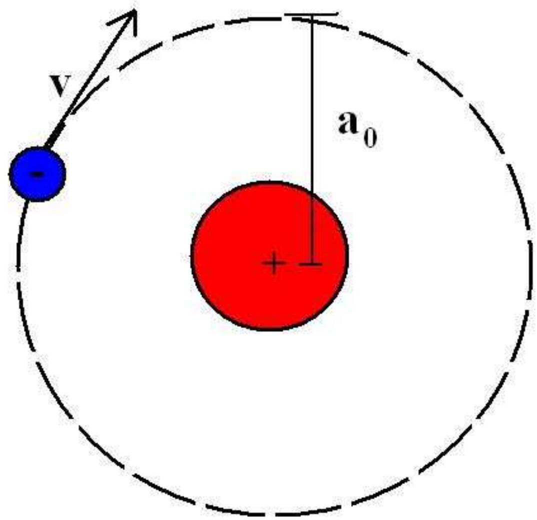
Gravity is always attractive, never repulsive!

Example

Comparing the gravitational and electric forces:

I've already mentionned that the gravitational force is much too weak to keep an electron in orbit around an atomic nucleus. Let's now show this explicitly, for hydrogen.

You all remember, I'm sure, from physics NYC that in the ground state of hydrogen, the electron is on a circular orbit of radius $r = a_0 = 0.0529$ nm. The speed of the electron is 2.19×10^6 m/s.



Example

Comparing the gravitational and electric forces:

The net force on an object of mass m in circular motion with speed v and radius r is $F = \dfrac{mv^2}{r}$. In this case, this is

$$F = \frac{m_e v^2}{a_0} = \frac{9.11 \times 10^{-31} \times (2.19 \times 10^6)^2}{0.0529 \times 10^{-9}} = 8.26 \times 10^{-8} \text{ N.}$$

The gravitational force between the electron and the proton is $F_g = G\dfrac{m_p m_e}{r^2} = 6.67 \times 10^{-11} \dfrac{1.67 \times 10^{-27} \times 9.11 \times 10^{-31}}{(0.0529 \times 10^{-9})^2} = 3.63 \times 10^{-47}$ N. This is way too small!!! The electric force, on the other hand, is

$$F_e = k_e \frac{|e||-e|}{r^2} = 8.99 \times 10^9 \times \frac{(1.602 \times 10^{-19})^2}{(0.0529 \times 10^{-9})^2} = 8.26 \times 10^{-8}$$

N, exactly the required amount.

Example

Comparing the gravitational and electric forces:

Imagine taking two 1 kg copper balls, and holding them 1 meter apart. What is the gravitational force between them? What is the electric force between them?

$$
\begin{aligned}
|\vec{F}_g| &= G\frac{m_1 m_2}{r^2} \\
&= 6.673 \times 10^{-11}\frac{1 \times 1}{1^2}\ \text{N} \\
&= 6.673 \times 10^{-11}\ \text{N}
\end{aligned}
$$

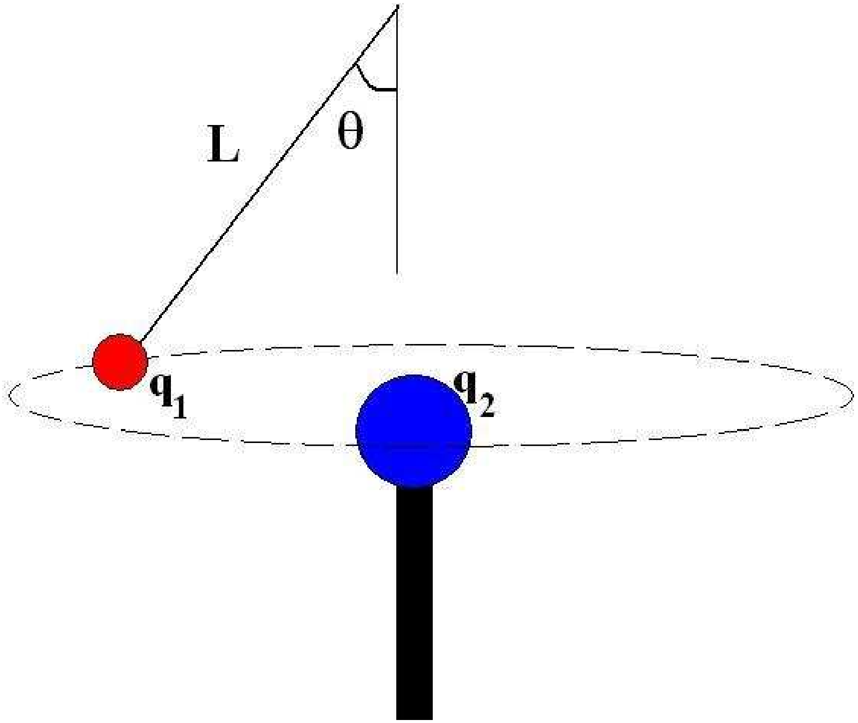
Since the spheres are both neutral, there is no electric force between them... What if we could somehow take some of the electrons from one sphere, and put them on the other so that each sphere now has a charge of $\pm$1 Coulomb?

Example

$$
\begin{aligned}
|\vec{F}_e| &= k_e \frac{q_1 q_2}{r^2} \\
&= 8.99 \times 10^9 \frac{1 \times 1}{1^2} \text{ N} \\
&= 8.99 \times 10^9 \text{ N}
\end{aligned}
$$

which is strong enough to lift the Great Pyramid!

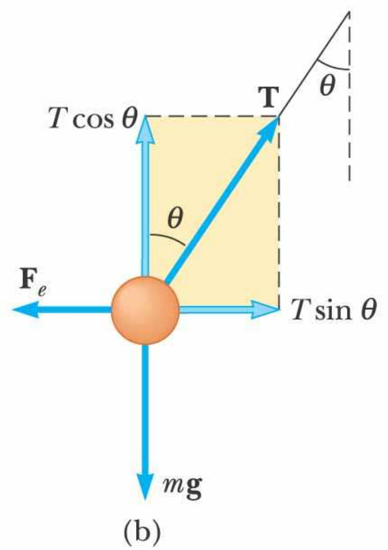
Example



A charge $q_1 = Q$ is placed on a pith ball of mass $m = 0.1$ g attached to a string of length $L = 15$ cm. An equal charge $q_2 = Q$ is placed on a small sphere. The pith ball orbits the sphere with a period $T = 0.75$ s and the string makes an angle of $\theta = 30^o$ with the vertical. What is the charge Q?

Example

Using a free body diagram and the fact that the net force on an object in cicular motion is $\dfrac{mv^2}{r}$ towards the center of the circle,



Example

$$\begin{aligned} T\sin\theta - F_e &= \frac{mv^2}{r} \\ T\cos\theta &= mg \\ \Rightarrow mg\tan\theta - k_e\frac{|q_1||q_2|}{r^2} &= \frac{mv^2}{r} \end{aligned}$$

The speed is the distance travelled $2\pi r$ during one period divided by the period T, so

$$\begin{aligned} mg\tan\theta - k_e\frac{|q_1||q_2|}{r^2} &= \frac{m(2\pi r/T)^2}{r} \\ mg\tan\theta - k_e\frac{|q_1||q_2|}{r^2} &= \frac{4\pi^2 rm}{T^2} \end{aligned}$$

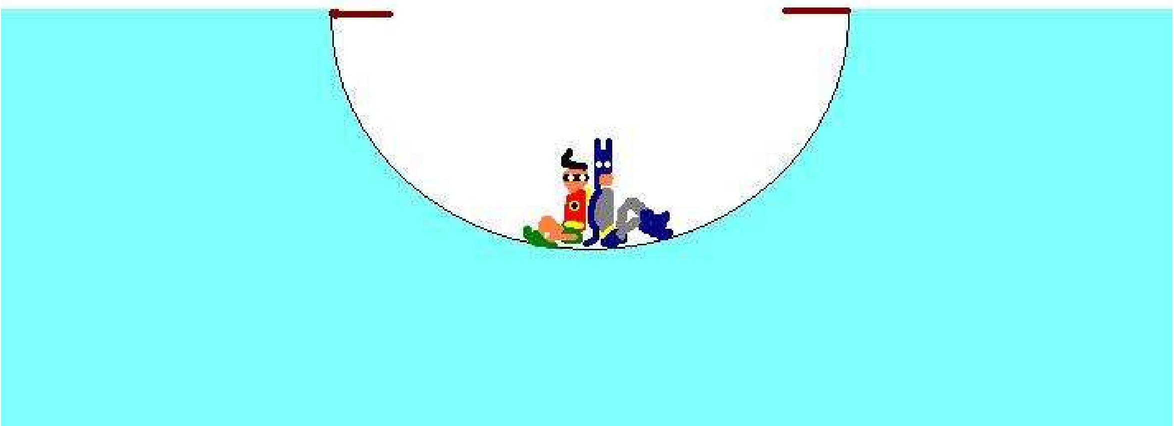
Example

$$\begin{aligned} mg\tan\theta - k_e\frac{|q_1||q_2|}{r^2} &= \frac{m(2\pi r/T)^2}{r} \\ mg\tan\theta - k_e\frac{|q_1||q_2|}{r^2} &= \frac{4\pi^2 rm}{T^2} \\ \Rightarrow |q_1||q_2| = Q^2 &= \frac{mr^2}{k_e}\left(g\tan\theta - \frac{4\pi^2 r}{T^2}\right) \end{aligned}$$

where $r = L\sin\theta$. Plugging in the values given in the statement, we find $Q = 5.00$ nC.

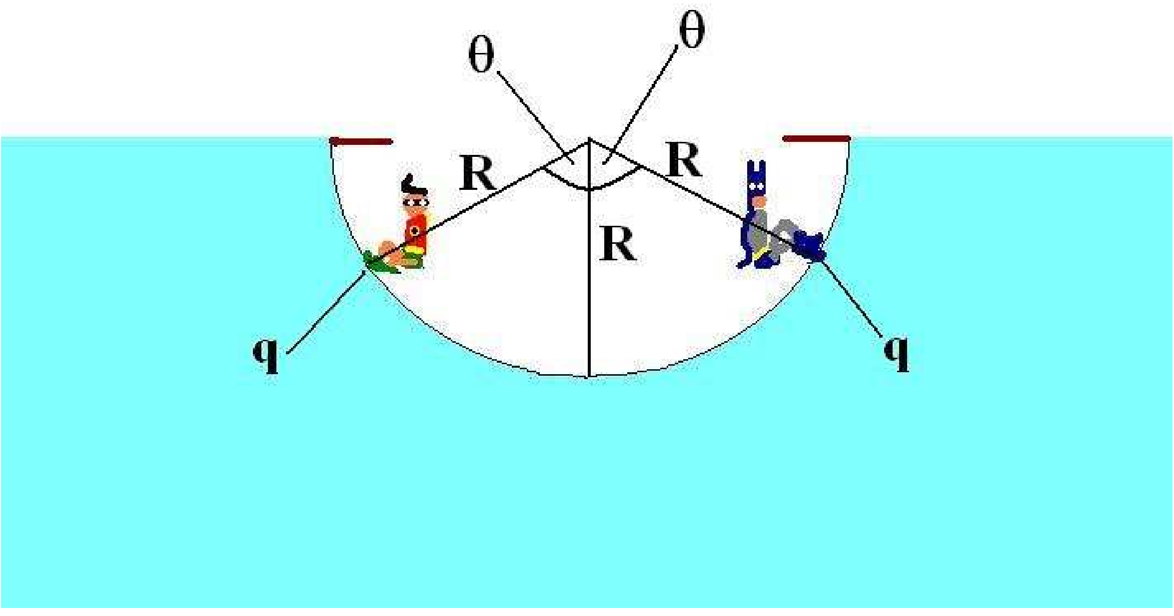
Example

After losing a fight to Mr. Freeze, Batman and Robin are stuck in the bottom of a perfectly frictionless, semi-circular hole of radius $R = 5$ m. In order to get out, they hatch the following plan. Each will vigourously rub his suit with his cape. This will oppositely charge the suit and cape. They will then get rid of the capes, leaving only their charged suits, which will repel each other.



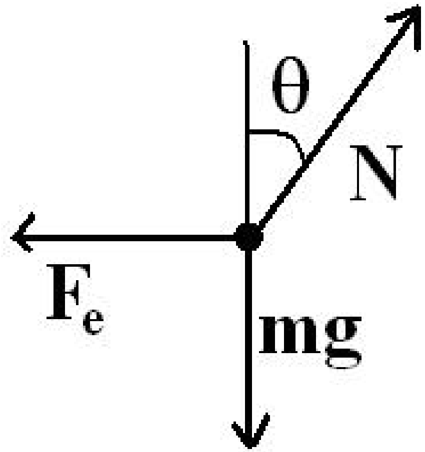
Example

This force must allow them to slide up the curved hole by 60^o, so they can grip conveniently placed branches and escape. Batman, as always, has a mass of 100 kg, while Robin has put on some pounds over the holidays and is now also 100 kg. What charge must they accumulate for this very straightforward and realistic plan to work?



Example

First, we make a free body diagram for either Batman or Robin (since both are the same).



For everything to be in equilibrium, we require

$$
\begin{aligned}
N\cos\theta - mg &= 0 \\
N\sin\theta - k_e\frac{q^2}{r^2} &= 0
\end{aligned}
$$

where $r = 2R\sin\theta$.

Example

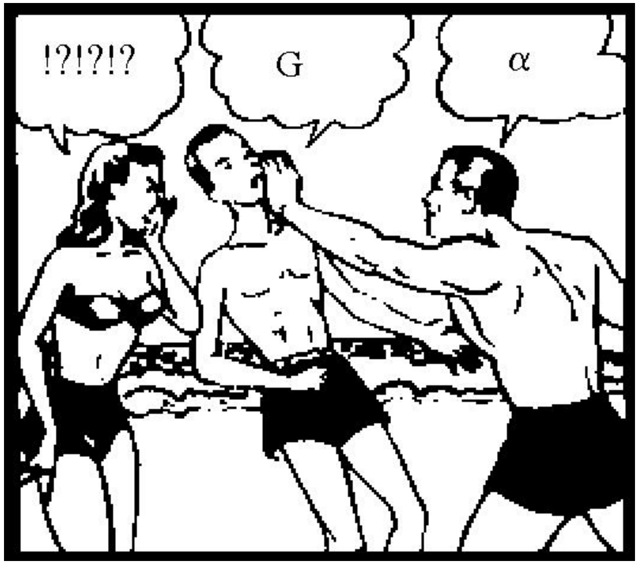
So

$$
\begin{aligned}
mg\tan\theta &= k_e \frac{q^2}{4R^2\sin^2\theta} \\
\Rightarrow q &= \sqrt{4R^2 \frac{mg}{k_e} \sin^2\theta \tan\theta}
\end{aligned}
$$

which, plugging in the required values, leads to $q = 3.76 \times 10^{-3}$ C, a *huge* charge! (But of course absolutely possible for heroes such as the caped crusaders...)

The concept of field

In physics NYA, you learned that there are two types of forces in the world:
<u>Contact forces and field forces.</u>



Contact forces imply actual *physical contact* between objects. Field forces imply *action at a distance*.

The concept of field

Question: What type of force is the electric force?

From the observations we made a few lectures ago, it is clear that the electric force is a *field force*.

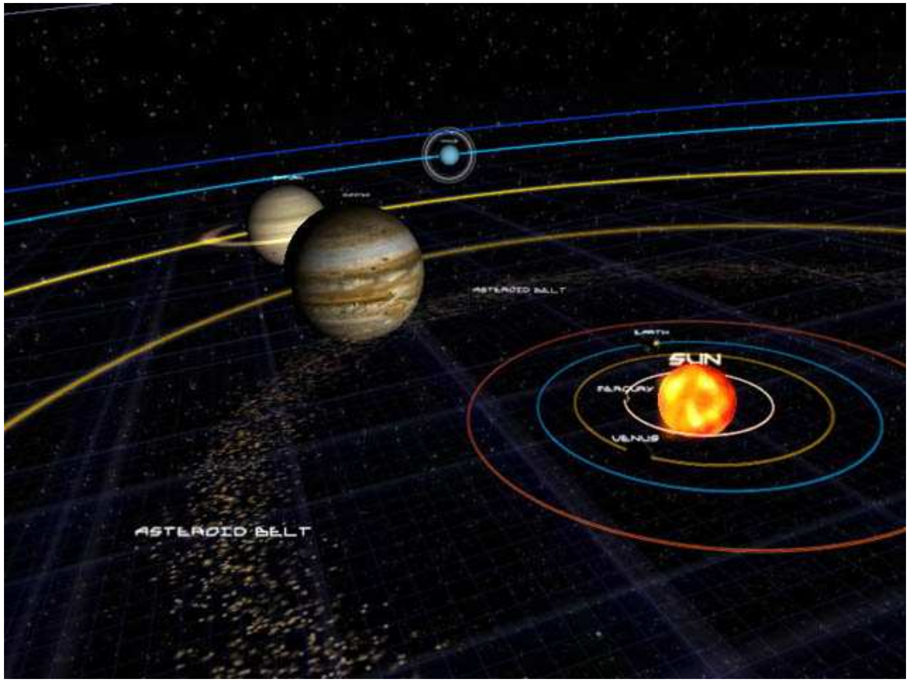
Question: Can you think of another field force?

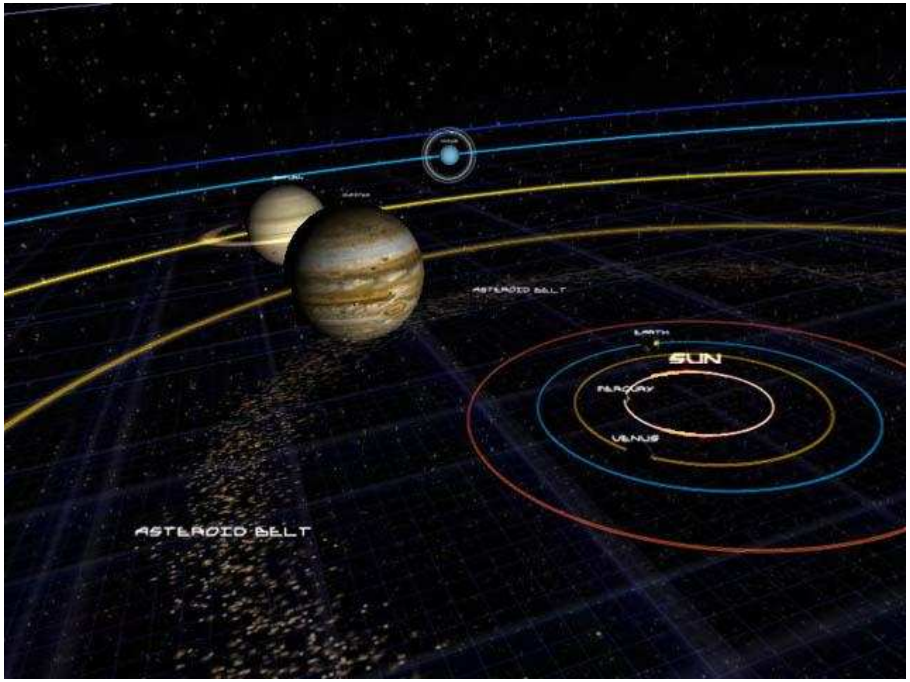
Gravity is also clearly a field force, as the Earth attracts you even when you are not touching it.

Question: does it make sense to you that two things can exert forces on each other without any physical contact between them? How can energy be exchanged between the two if no physical entity transmits the energy from one to the other?
Hmmm...

The concept of field

To illustrate this, imagine the Sun were to suddenly disappear. Would the planets immediately feel this? Would someone on Earth immediately *see* the Sun disappear?





The concept of field

To illustrate this, imagine the Sun were to suddenly disappear. Would the planets immediately feel this? Would someone on Earth immediately *see* the Sun disappear?

Light travels at a speed $c = 3 \times 10^8$ m/s, so someone on Earth would definitely not *see* that the Sun is gone right away. (It would actually take about 8 minutes for the last light the Sun emitted to reach the Earth.)

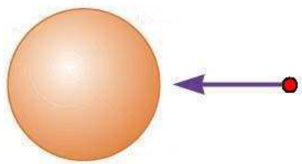
But in physics NYC, you learned that *nothing can travel faster than light!* This means there is no way the planets could *feel* the effect of the Sun disappearing instantaneously.

The concept of field

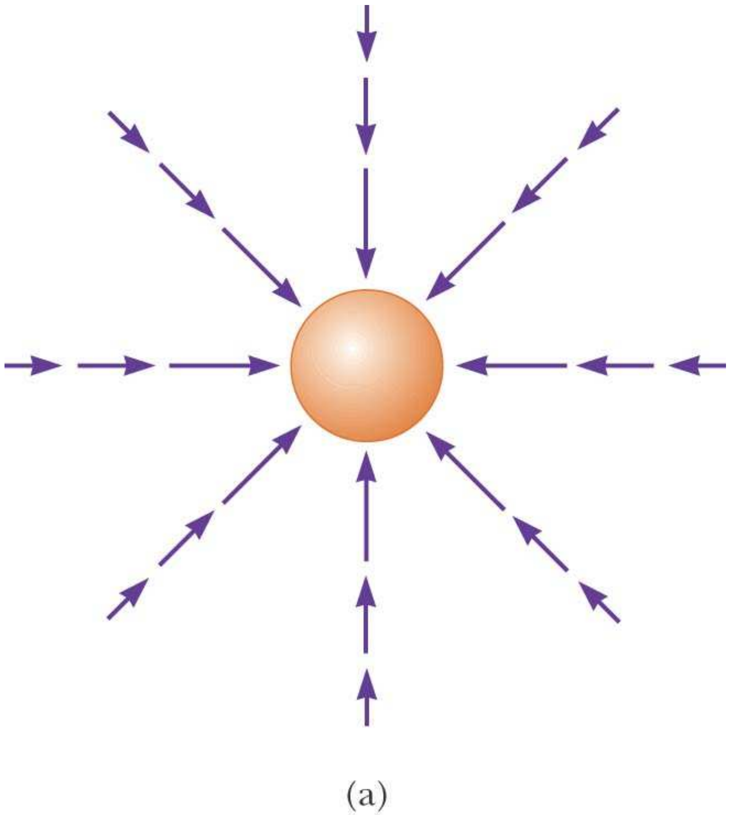
- So there really is no such thing as *action at a distance*.
- When a force acts between objects that are not in physical contact, there must be a *physical entity* actually exerting the force.
- This physical entity is called a *field*.
- The gravitational force is exerted by the gravitational field.
- The electric force is exerted by the *electric field*.

Evidence for the electric field

We know the electric field is there from the effect it has on a charge. For example, if we deposit charges on a metal sphere, we can take a small charge and move it around the sphere measuring the force at different places. Let's do this, drawing the force vector at every location.

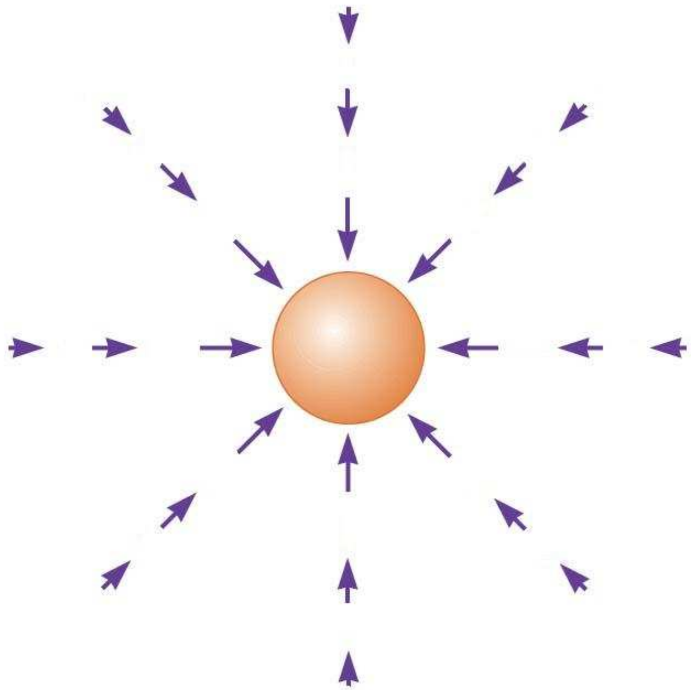


The concept of field



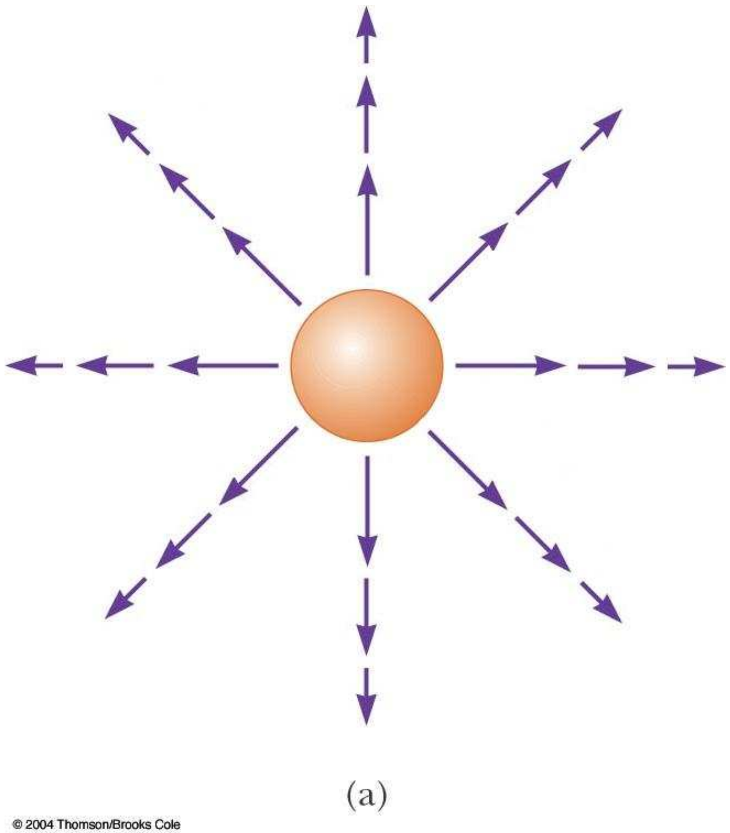
So at *every single point* in space, we have a *vector* indicating the force that our charge felt from the sphere when placed at that point.

The concept of field



If we had put less charge on the sphere, or used a smaller charge to find the force near the sphere, the measured force would be weaker.

The concept of field



If we had put opposite charge on the sphere, or used opposite charge to find the force near the sphere, the measured force would be in the opposite direction.

The concept of field

- We will denote the electric field as $\vec{E}$. It is *created by the charged sphere*; the charged sphere is *the source* of the electric field.
- The effect of the field *is felt by the charge* q_0 we place in the field; we call this charge the *test charge*.
- The electric field is there *whether or not a test charge is present*.
- The force felt by the test charge q_0 depends on
  - The value of the test charge
  - The magnitude and direction of the electric field $\vec{E}$

$$\boxed{\vec{F}_e = q_0 \vec{E}}$$

The concept of field

$$\boxed{\vec{F}_e = q_0\vec{E}}$$

- An electric field is associated with any charge.
- It extends all over space.
- The electric field is a vector which, at any given position, points in the direction in which the force felt by a *positive* test charge placed in the field at that same position would point.
- The magnitude of the electric field represents the force that a 1 C charge would feel when placed in the field.
- Looking at the equation, we see that the units of $\vec{E}$ are N/C

Electric field for a point charge

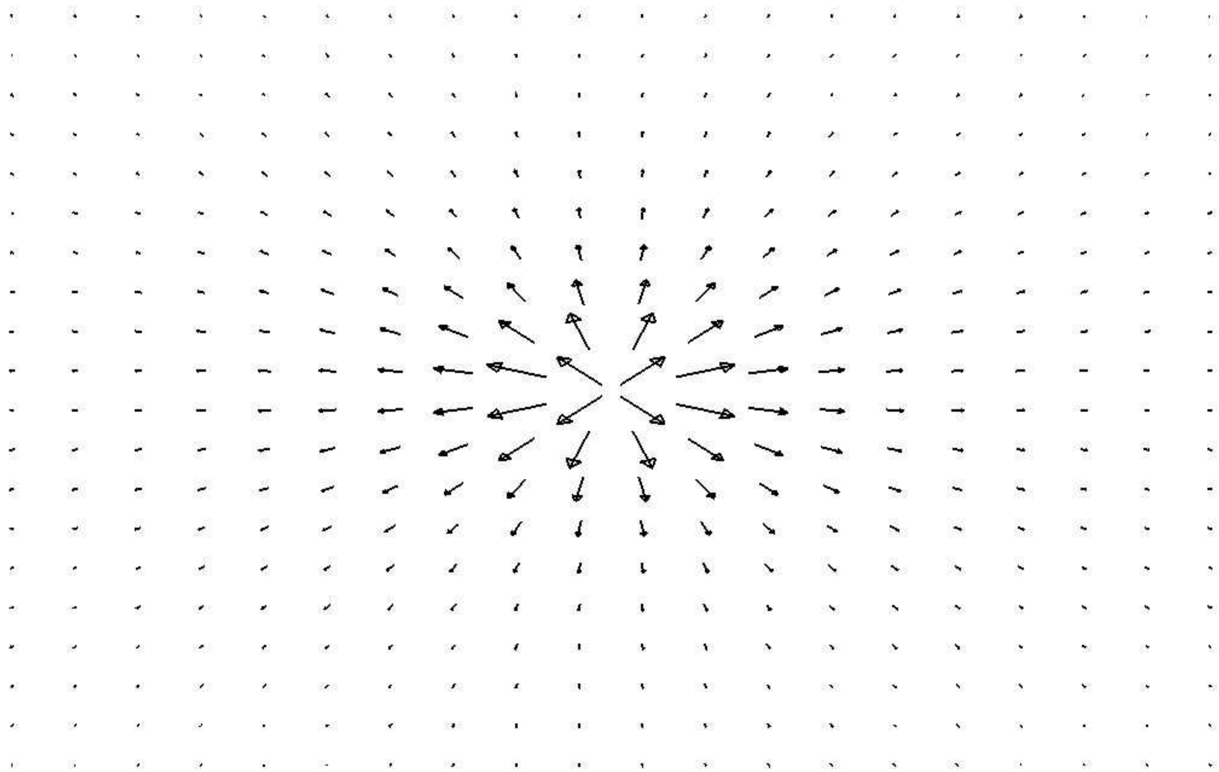
We can turn the relation $\vec{F}_e = q_0 \vec{E}$ around, and *define* the electric field to be $\vec{E} = \dfrac{\vec{F}}{q_0}$.

Since the force between a point charge q and a point charge q_0 is $|\vec{F}_e| = k_e \dfrac{|q||q_0|}{r^2}$, this means that the electric field associated with the point charge q is

$$\boxed{\vec{E} = k_e \frac{q}{r^2} \hat{r}}$$

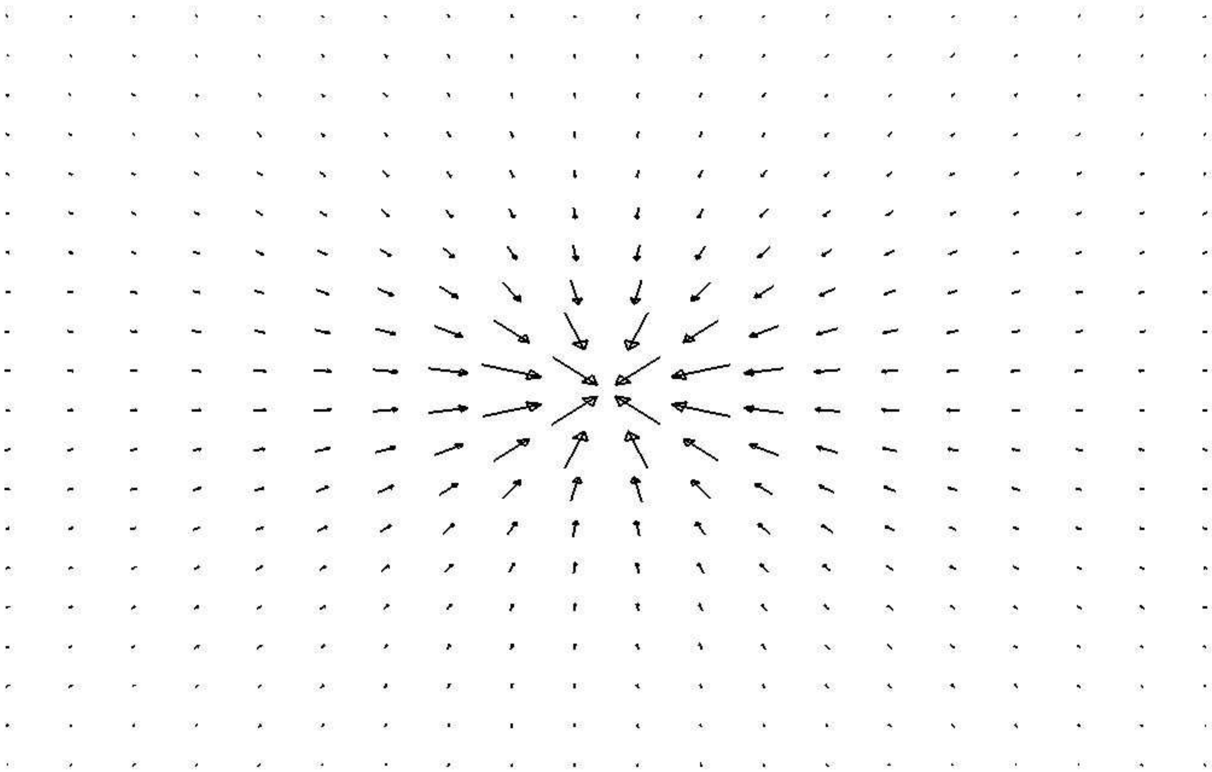
(where $\hat{r}$ is a unit vector always pointing directly away from the charge q).

The concept of field



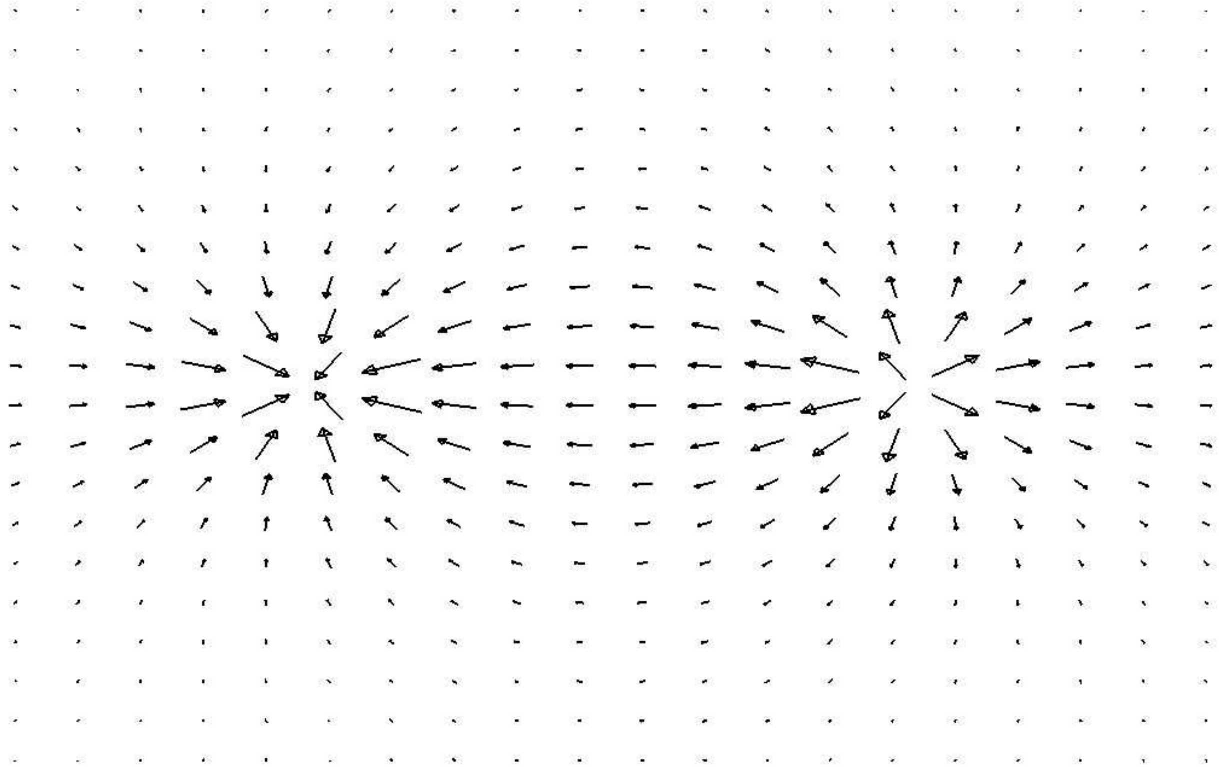
Electric field for a positive point charge. The force felt by a positive test charge is *away* from the point charge everywhere, and the magnitude decreases with the square of the distance from the point charge.

The concept of field



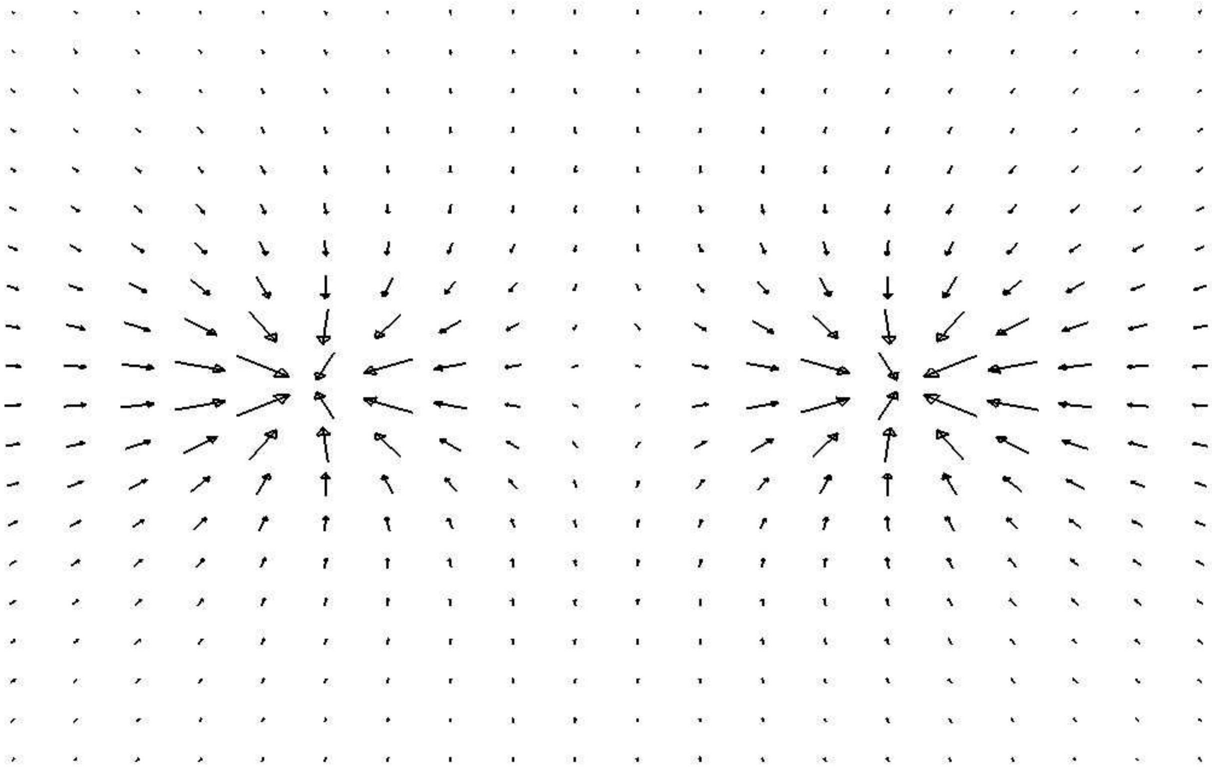
Electric field for a negative point charge. The force felt by a positive test charge is *towards* the point charge everywhere, and the magnitude decreases with the square of the distance from the point charge.

The concept of field



Electric fields of a positive and a negative point charge, adding up through superposition. Just like we add up the individual forces when multiple charges are present, if two or more charges create electric fields, the net field is the sum of the individual fields.

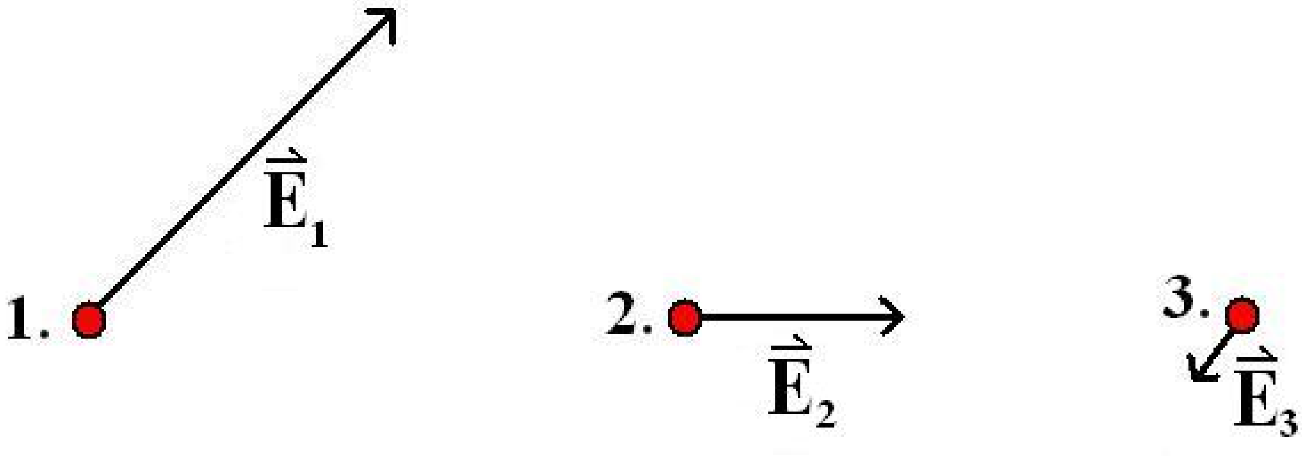
The concept of field



Electric field for two negative point charge.

Examples

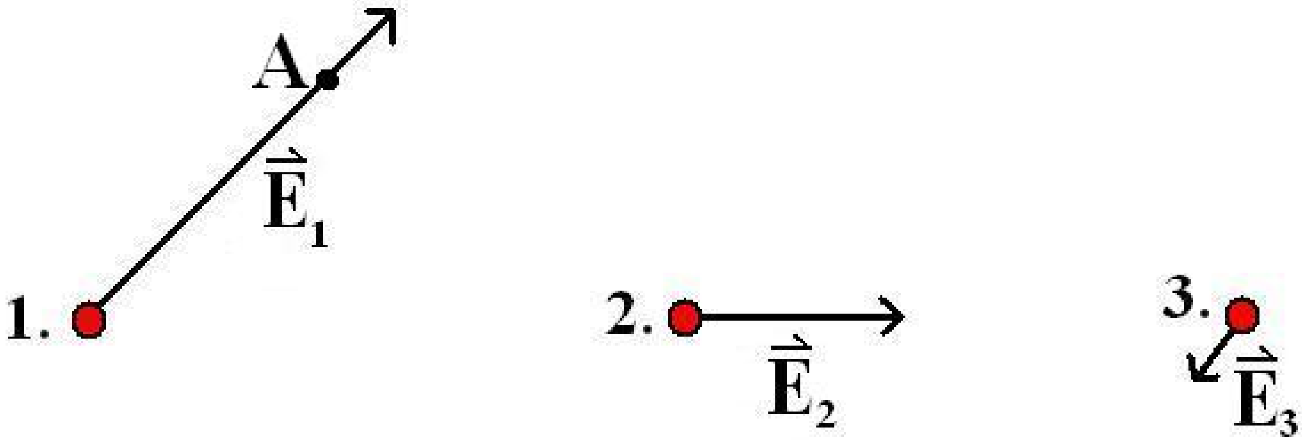
Here, we have pictured the force felt by a positive test charge q placed at different points in space. Draw the corresponding electric field vectors at points 1, 2 and 3.



The electric field is in the same direction and proportional to the force felt by a positive test charge, so this is a trivial task.

Examples

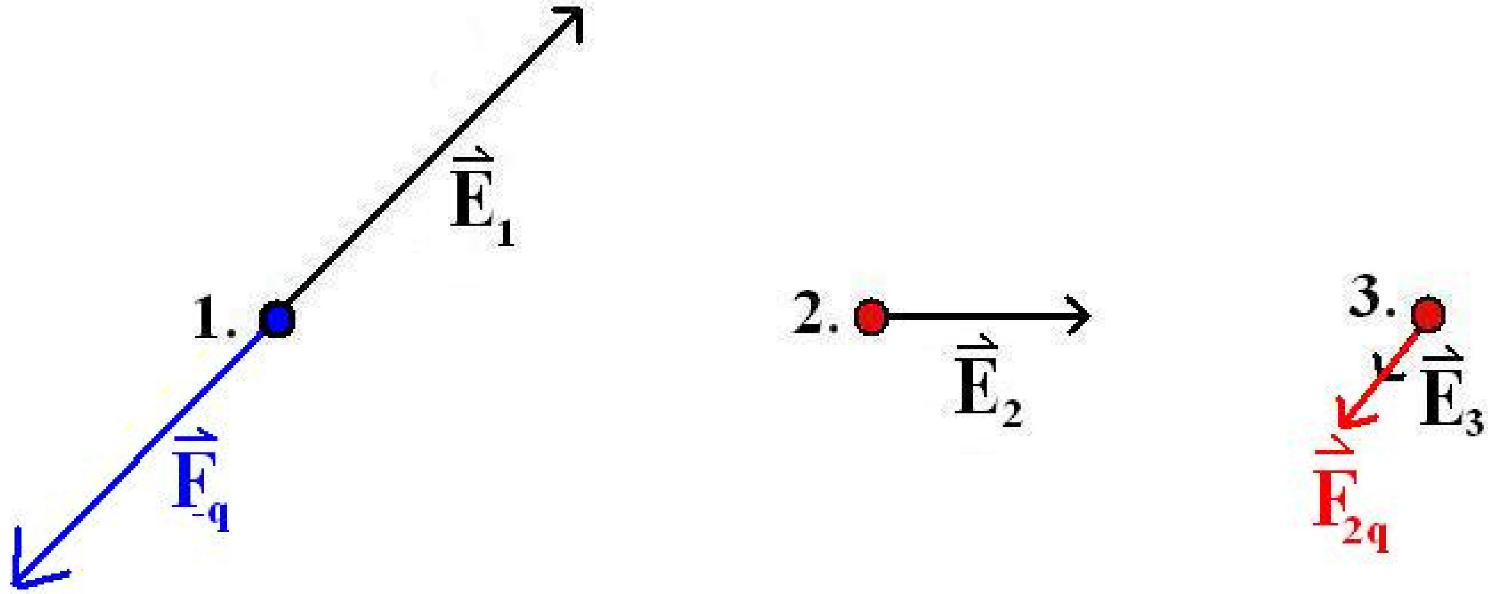
Now that we know the electric field at points 1, 2 and 3, can you draw the force vector acting on a positively charged particle (of unit charge) placed at point A?



No. Even though point A happens to be placed on the vector corresponding to $\vec{E}$ at point 1, we have no idea what the electric field is *at point A!* The $\vec{E}$ field at point A is a completely different vector from the $\vec{E}$ field at point 1.

Examples

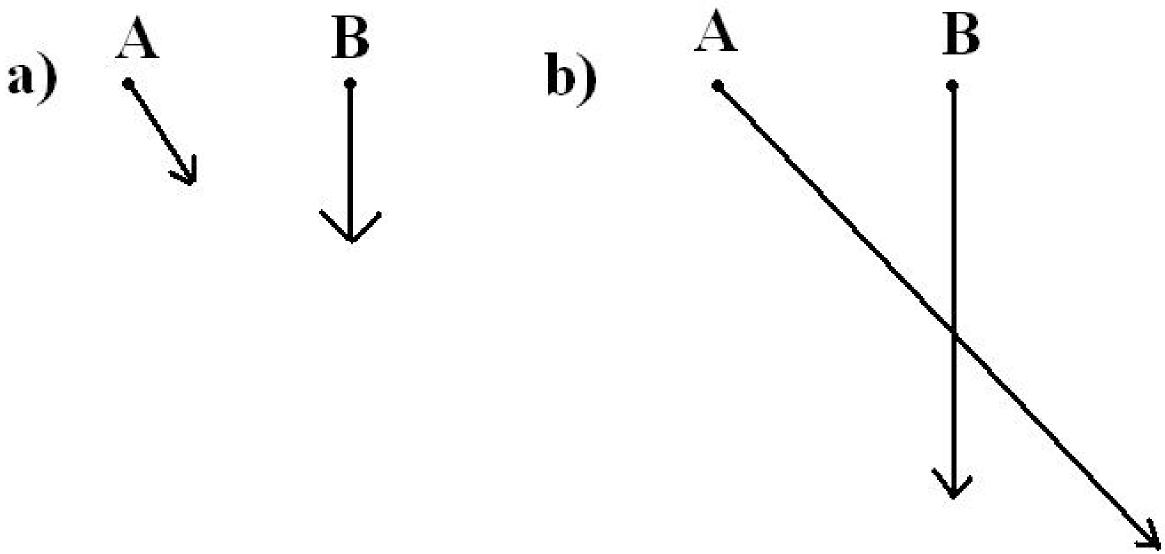
Now, draw the force vectors for a charge $-q$ placed at point 1, and a charge $2q$ placed at point three.



The force on the negative charge is in the opposite direction of the field vector, but has the same magnitude. The force on the positive charge is in the same direction as the field vector, but has twice the magnitude, since we are talking about a charge $2q$.

Examples

Which of these correspond to possible electric field configurations?



Both are possible. You might be thinking that the field is pointing in two different directions at the point where the arrows cross on the right, but that's not the case. These arrows represent the electric field *at points A and B*. The field at the points where the vectors happen to cross is another vector altogether, which isn't pictured here.

What to read for next lecture

- 23.6, 23.7